Review of Logic and Rules

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General Caveat

If you think that your paper is vacuous,

use the first-order functional calculus.

It then becomes logic,

and as if by magic,

the obvious is hailed as miraculous.

Paul Halmos c.1967

A great book:

Introduction to Logic by Patrick Suppes

- If Alfred studies then he receives good grades.
- If Alfred does not study then he enjoys college.
- If Alfred does not receive good grades then he does not enjoy college.

Therefore Alfred receives good grades.

Do we believe this argument?

if so, why?

if not, why not?

Logic is the study of arguments

An argument is a sequence of statements beginning with premises and ending with a conclusion.

The conclusion is said to follow from (or is supported by) the premises.

- Arguments may be good or bad (convincing or not convincing) A good argument is called valid.
- A valid form of argument is one that never allows the deduction of a false conclusion from true premises.
- An invalid form of argument is one that sometimes allows the deduction of a false conclusion from true premises.

A Valid Argument

If Socrates is human then Socrates is mortal.

Socrates is human.

Socrates is mortal.

An Invalid Argument

If Socrates is human then Socrates is mortal.

Socrates is mortal.

Socrates is human.

I have a dog named Socrates!

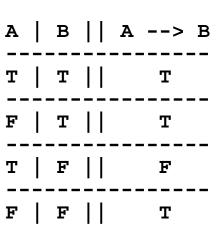
Sentential Connectives Truth functions

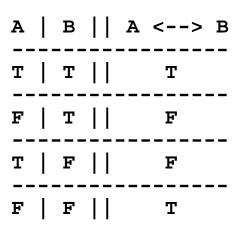
Implication: a --> b If a then b. a implies b. a only if b. Negation: -a Not a. It is not the case that a. Conjunction: a & b a and b. Disjunction: a V b a or b. Equivalence: a <--> b a if and only if b.

Truth Tables define Truth Functions

A | B || A & B т | т || Т - - -F | T || F T | F || F F | F | | F

	•				в
т	I	т		т	
		Т			
Т	· ·	F		Т	
F		F		 F	





Analysis of arguments

by sentential interpretation.

Primitive sentences

S = Alfred studies.E = Alfred enjoys college. G = Alfred receives good grades.

Premises

A = S> G B = -S> E C = -G> -E									
S	E	G	A	в	C				
т	Т	Т	Т	Т	Т				
т	т	F	F	т	F				
т	F	т	т	т	Т				
т	F	F	F	т	Т				
F	т	т	F	т	Т				
F	т	F	F	т	F				
F	F	т	F	F	Т				
F	F	F	Т	F	т				

Note that for every row where the conclusion G is false, at least one of the premises (A, B, or C) is false.

Sentential Derivation

 If Alfred studies then he receives good grades.

PREMISE

2. If Alfred does not study then he enjoys college.

PREMISE

3. If Alfred does not receive good grades then he does not enjoy college.

PREMISE

4. Alfred does not receive good grades.

PREMISE (hypothetical)

5. Alfred does not enjoy college.

MP 3,4

6. Alfred studies.

MT 2,5; NE

7. Alfred does not study.

MT 1,4

8. Alfred receives good grades.

RAA 4,6,7

QED

Sentential Derivation with Dependencies

The Suppes Formalism

 $\{1\}$ $\{2\}$ 1. All y [(greek y) --> (human y)] Premise 2. All x [(human x) --> (mortal x)] Premise 3. (greek *G) Premise {3} (US 1 *G y) 4. (greek *G) --> (human *G) {1} (MP 4 3) $\{1 \ 3\}$ 5. (human *G) $(US 2 *G x) \{2\}$ 6. (human *G) --> (mortal *G) $(MP 6 5) \{123\}$ 7. (mortal *G) 8. (greek *G) --> (mortal *G) $(CP 7 3) \{1 2\}$ 9. All z [(greek z) --> (mortal z)] (UG 8 z *G) {1 2}